Curtin University, Semester 2, 2026

ECOM5005

Tutorial 5 Detailed Solutions

Probability Distributions - classroom-ready worked solutions

Learning Outcomes

1. Recognise and apply the properties of a probability distribution.
2. Calculate the expected value and variance of a probability distribution.
3. Calculate average return, and measure risk associated with various investment proposals.
4. Calculate probabilities from binomial, Poisson and normal distributions.
5. Use the binomial, Poisson and normal distributions to solve business problems.

PART 1 - Detailed worked solutions

Question 1. Life-insurance survival probabilities

Question

An agent sells life insurance policies to five equally aged, healthy people. The probability that each person survives for at least 30 years is p = 2/3. Find: (a) the probability all five survive; and (b) the probability at least three survive.

Detailed Solution

Step 1: Identify the distribution. Let X be the number of the five people who are still living after 30 years. Under the model in the question, X follows a binomial distribution with n = 5 and p = 2/3.
Step 2: Part (a) - all five are still living. This is X = 5.

P(X = 5) = C(5,5)(2/3)^5(1/3)^0 = (2/3)^5 = 32/243 = 0.131687.

So the probability is approximately 0.1317, or 13.17%.

Step 3: Part (b) - at least three are still living. “At least 3” means X = 3, 4 or 5, so add the three mutually exclusive probabilities.

P(X ≥ 3) = P(X=3)+P(X=4)+P(X=5) = 0.790123.

Excel check: =BINOM.DIST(5,5,2/3,TRUE)-BINOM.DIST(2,5,2/3,TRUE) is unnecessary; the simplest cumulative form is =1-BINOM.DIST(2,5,2/3,TRUE).

Answer Summary

(a) 0.1317 (13.17%).
(b) 0.7901 (79.01%).

Question 2. Expected value of the die game

Question

A fair die is rolled. Prime outcomes produce a gain equal to the number rolled × $100; non-prime outcomes produce an equal-sized loss. Construct the probability distribution and calculate the expected value.

Detailed Solution

Step 1: Classify the six equally likely die outcomes. The prime numbers on a standard die are 2, 3 and 5. The non-prime outcomes are 1, 4 and 6.
Step 2: Translate each die outcome into a monetary payoff. Because the die is fair, every payoff has probability 1/6.
Step 3: Expected value is the probability-weighted average: $E(X) = \Sigma\, x_i P(X=x_i)$.

Payoff x	P(X=x)	xP(X=x)
-$100	1/6	-$16.67
+$200	1/6	+$33.33
+$300	1/6	+$50.00
-$400	1/6	-$66.67
+$500	1/6	+$83.33
-$600	1/6	-$100.00

$$E(X) = (-100 + 200 + 300 - 400 + 500 - 600) / 6 = -100/6 = -\$16.67$$

Interpretation

The expected value is negative. In the long run, repeated play under these rules would lose about $16.67 per game on average. The expected value is a long-run average, not a guaranteed payoff from one roll.

Question 3. Poisson calls to a support desk

Detailed Solution

The desk receives 2 calls every 3 minutes. A 9-minute interval is three times as long, so the Poisson mean is $\lambda = 2 \times 3 = 6$ calls.
We need $P(X \le 4)$. In Excel enter =POISSON.DIST(4,6,TRUE).
The result is $P(X \le 4) = 0.2850565 \approx 0.2851$.

Answer Summary

Probability of 4 or fewer calls in 9 minutes = 0.2851 (28.51%).

Question 4. Normal parking-time probability

Detailed Solution

Let X be parking-search time in minutes. The question states X ~ Normal(μ = 3.5, σ = 1). We need P(X < 3).
Standardised value: z = (3 - 3.5)/1 = -0.5.
Excel method: =NORM.DIST(3,3.5,1,TRUE).
P(X < 3) = 0.3085375 ≈ 0.3085.

Interpretation

About 30.85% of students are expected to find a parking space in under 3 minutes under the stated normal model.

Extension problems - detailed solutions

Extension Problem A. EV battery quality-control audit

Model choice

This is hypergeometric because the population is finite (N = 30), there are A = 5 defective "successes", n = 6 packs are sampled, and sampling is without replacement. Therefore the draws are dependent.

Detailed Solution

Exactly one defective pack:
P(X=1) = C(5,1)C(25,5) / C(30,6) = 0.4473917 ≈ 0.4474.
At least one defective pack is easiest by the complement rule:
P(X≥1) = 1 - P(X=0) = 1 - C(25,6)/C(30,6) = 0.7017389 ≈ 0.7017.
Expected number: E(X) = n(A/N) = 6(5/30) = 1.0000.
Standard deviation: σ = √[n(A/N)(1-A/N)((N-n)/(N-1))] = 0.8305.
Excel checks: =HYPGEOM.DIST(1,6,5,30,FALSE) and =1-HYPGEOM.DIST(0,6,5,30,FALSE).

Interpretation

There is about a 70.17% chance that the six-pack audit finds at least one defective battery. The expected count is one defective pack, although an individual sample can contain 0, 1, 2, ... defectives.

Extension Problem B. Same-day grocery delivery service level

Detailed Solution

Let X ~ Normal(32, 6).
(a) Voucher probability: P(X > 45) = 1 - P(X ≤ 45) = 1 - NORM.DIST(45,32,6,TRUE) = 0.01513 ≈ 1.51%.
(b) P(28 < X < 38) = NORM.DIST(38,32,6,TRUE) - NORM.DIST(28,32,6,TRUE) = 0.58885 ≈ 58.89%.
(c) A promise met by 95% of orders is the 95th percentile: NORM.INV(0.95,32,6) = 41.8691 minutes, so about 41.9 minutes.
(d) Excel functions: NORM.DIST for probabilities and NORM.INV for percentile cut-offs.

Business interpretation

Only about 1.5% of deliveries are expected to exceed 45 minutes. A promised time of roughly 42 minutes corresponds to a 95% service level under the model.

Extension Problem C. Cybersecurity alert surge

Detailed Solution

The hourly Poisson rate is 3.2. For 90 minutes, $\lambda = 3.2 \times 1.5 = 4.8$.
(a) $P(X \geq 7) = 1 - P(X \leq 6) = 1$ - POISSON.DIST(6,4.8,TRUE) = 0.20920 ≈ 20.92%.

(b) $P(3 \leq X \leq 6) = P(X \leq 6) - P(X \leq 2) = 0.64827 \approx 64.83\%$.

(c) For a Poisson variable, mean = $\lambda = 4.8$ and standard deviation = $\sqrt{\lambda} = 2.1909$ alerts.

(d) A 20.9% probability of seven or more alerts is not rare; managers could use this as evidence for surge staffing, automated triage or an escalation trigger during 90-minute monitoring windows.

PART 2 - Data exercise: super-detailed Excel solution

What is in the workbook

The stock_price_2017.xlsx file contains 250 trading-date rows from 3 January 2017 to 29 December 2017. Columns B and C contain AAPL and MSFT prices. Columns D and E contain daily simple returns; the first price row has no return because there is no previous trading day. Therefore there are 249 usable daily-return observations for each stock.
This solution uses rows 3:251 for the return calculations.

Step 0 - Save a working copy before calculating anything

- Open stock_price_2017.xlsx in Excel.
- Choose File > Save As and save a copy such as Tutorial5_StockAnalysis_YourName.xlsx.
- Work in the copy so that you can always return to the original data if a formula is accidentally overwritten.

Step 1 - Check the return formulas

The simple return from one trading day to the next is (new price - old price) / old price. If the return columns are already populated, check a few rows rather than replacing them.

Exact Excel formulas

In D3 enter: =(B3-B2)/B2
In E3 enter: =(C3-C2)/C2

Press Enter. Click D3 again and drag the fill handle down to D251. Repeat for E3:E251.

Select D3:E251, then Home > Number > Percentage and display 2 decimal places for easy reading.

Step 2 - Compute descriptive statistics for AAPL and MSFT returns

Recommended method for beginners: use Excel's Data Analysis ToolPak so that Excel creates a complete descriptive-statistics table.

- If Data Analysis is not visible on the Data tab: File > Options > Add-ins > Manage Excel Add-ins > Go > tick Analysis ToolPak > OK.
- Go to Data > Data Analysis > Descriptive Statistics.
- For AAPL, set Input Range to $D$3:$D$251, choose Columns, tick Summary statistics, choose an empty Output Range, then click OK.
- Repeat using $E$3:$E$251 for MSFT.
- Because the input range does not include a heading, do not tick "Labels in first row".

The checked final results are:

Statistic	AAPL return	MSFT return
Mean	0.1575%	0.1299%
Standard Error	0.0709%	0.0592%
Median	0.0667%	0.0395%
Mode	No unique mode	0.0000%
Standard Deviation	1.1181%	0.9342%
Sample Variance	0.000125015	0.000087278
Kurtosis	4.9780	8.8840
Skewness	0.5331	0.8688
Range	9.9757%	10.1859%
Minimum	-3.8777%	-3.7740%
Maximum	6.0981%	6.4119%
Sum	39.2084%	32.3490%
Count	249	249

How to read this table

AAPL has the higher average daily return (0.1575% versus 0.1299%), but AAPL also has the higher daily standard deviation (1.1181% versus 0.9342%). In this sample, AAPL therefore delivered slightly more average daily return together with greater stand-alone volatility.
Both return series are positively skewed and have positive excess kurtosis, indicating right-skewness and heavier tails than a normal distribution.

Step 3 - Compute covariance of the two return series

Exact Excel formula

Click an empty cell and enter: =COVARIANCE.S(D3:D251,E3:E251)
Press Enter. The result should be 0.0000457164.

The covariance is positive, so AAPL and MSFT returns tended to move in the same direction during 2017. For easier interpretation, an optional correlation check is =CORREL(D3:D251,E3:E251) = 0.4377.

Step 4 - Compute the equal-weight portfolio mean

Formula and result

For equal weights, wAAPL = 0.5 and wMSFT = 0.5.
E(P) = 0.5E(AAPL) + 0.5E(MSFT).

Excel: =0.5*AVERAGE(D3:D251)+0.5*AVERAGE(E3:E251)

Result: 0.0014368967 = 0.1437% per trading day.

Step 5 - Compute portfolio risk using variance, covariance and weights

Formula and result

Portfolio variance = $wA^2Var(AAPL) + wM^2Var(MSFT) + 2wAwM\ Cov(AAPL,MSFT)$.
Excel: =0.5^2*VAR.S(D3:D251)+0.5^2*VAR.S(E3:E251)+2*0.5*0.5*COVARIANCE.S(D3:D251,E3:E251)

Portfolio variance = 0.0000759315.

Portfolio risk is the square root: =SQRT(portfolio_variance) = 0.0087138665 = 0.8714% per trading day.

Step 6 - Create the portfolio-return series and verify the answer

- In F1 type Equal-weight Portfolio Return.
- In F3 enter =0.5*D3+0.5*E3 and press Enter.
- Fill F3 down through F251.
- Check the mean with =AVERAGE(F3:F251). You should obtain 0.1437%.
- Check the risk with =STDEV.S(F3:F251). You should obtain 0.8714%.
- These direct calculations must match Steps 4 and 5. If they do not, check row ranges and weights.

Investment	Mean daily return	Daily risk (SD)
AAPL	0.1575%	1.1181%
MSFT	0.1299%	0.9342%
50/50 portfolio	0.1437%	0.8714%

Step 7 - Create Figure 1: AAPL and MSFT prices across 2017

- Select A1:C251.
- Go to Insert > Line or Area Chart > 2-D Line.
- Change the chart title to AAPL and MSFT Closing Prices in 2017.
- Use Chart Design > Add Chart Element > Axis Titles and label the vertical axis Price ($).
- Keep the legend so students can distinguish the two stocks.

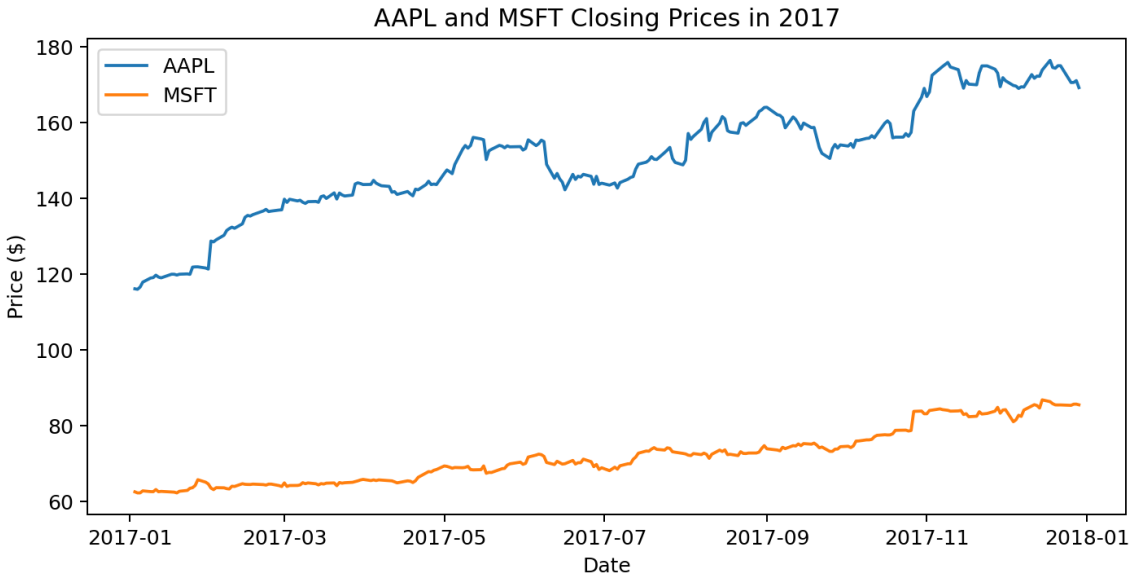

Figure 1: AAPL and MSFT closing prices during 2017

Step 8 - Create Figure 2: scatter plot of AAPL versus MSFT returns

- Select D3:E251.
- Go to Insert > Scatter (X, Y) > Scatter with only markers.
- If Excel assigns the axes incorrectly, right-click the chart > Select Data > Edit series. Set X values to D3:D251 and Y values to E3:E251.
- Add axis titles: AAPL daily return (%) on the horizontal axis and MSFT daily return (%) on the vertical axis.
- Right-click a point > Add Trendline > Linear. A positively sloped line is consistent with the positive covariance.

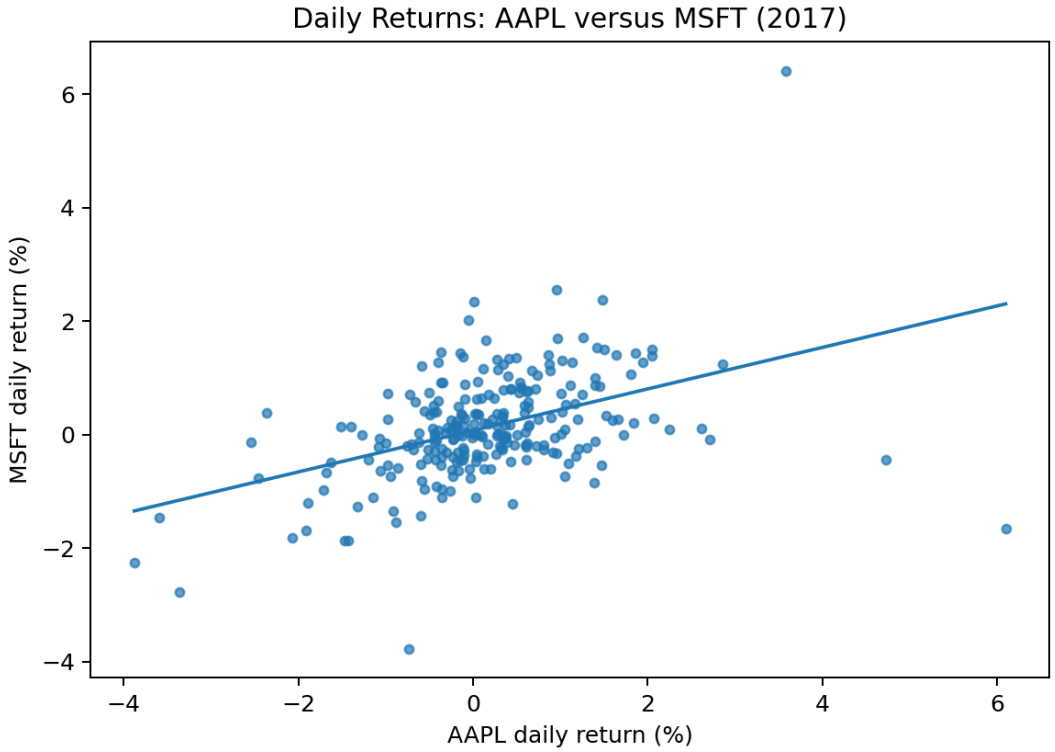

Figure 2: The return relationship is positive but far from perfect; the sample correlation is approximately 0.438

Step 9 - Create Figure 3: histogram of the equal-weight portfolio return

- Select F3:F251.
- Go to Insert > Insert Statistic Chart > Histogram.
- Give the chart the title Equal-Weight Portfolio Daily Returns (2017).
- If desired, right-click the horizontal axis > Format Axis and set a sensible bin width. The exact appearance can vary slightly with the chosen bin settings.

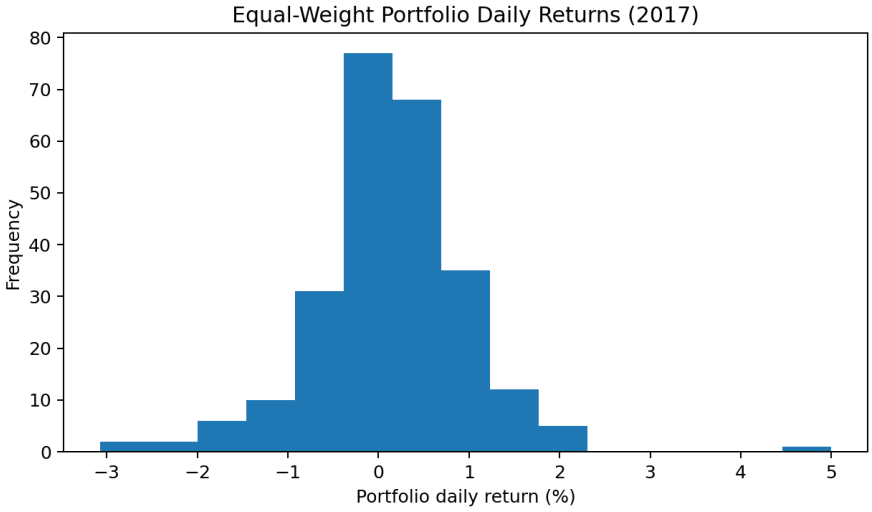

Figure 3: Distribution of the 249 equal-weight daily portfolio returns

Step 10 - Final interpretation for Question 5(h)

Model interpretation

AAPL produced a slightly higher average daily return than MSFT in this 2017 sample, but AAPL also had higher stand-alone volatility.
The covariance is positive, so the two stocks tended to move together, but the relationship was not perfect (correlation ≈ 0.438).

The 50/50 portfolio has an average daily return of about 0.1437%, which lies between the two individual means, while its daily standard deviation is only about 0.8714% - lower than the risk of either stock individually.

This reduction in volatility illustrates diversification: combining assets that are not perfectly positively correlated can lower portfolio risk.

Final numerical answer summary

AAPL mean = 0.1575%; AAPL SD = 1.1181%.
MSFT mean = 0.1299%; MSFT SD = 0.9342%.

Sample covariance = 0.0000457164; optional sample correlation = 0.4377.

Equal-weight portfolio mean = 0.1437%.

Equal-weight portfolio variance = 0.0000759315.

Equal-weight portfolio risk (SD) = 0.8714%.

PART 3 - Pearson MyLab

Please logon to MyLab and complete the following tasks.

1. Complete the Homework #3 questions.

Note: If you are having issues with obtaining access to MyLab, please re-read the instructions (available in the welcome topic). If you are still unsuccessful, please send an email to the teaching team explaining the issue. It is important that you complete MyLab tasks on a weekly basis since the final e-test in this unit will be completed using MyLab.